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Mathematical modeling of dynamics of the system “ship-waterways” behavior

Abstract: The processes of the movement of the ship in conditions of considerable compression in some sections of the waterway are considered in the article. Author developed a mathematical process based on the Navier-Stokes equations. Paper presents the results of the modeling process on the example of Tchaikovsky’s gateway. On the basis of mathematical modeling it was found that the erosion speed under the hull, substantially exceeds the speed of the ship and, contrary to the popular view, has an inverse dependence on vessel speed.

Keywords: system “ship-waterway” behavior, Navier-Stokes equations, turbulence model, „bank” effects

1. Introduction

At all times people tried to organize their home on the banks of large rivers. On the one hand, this provided water use and fishing, and on the other, transport communications with other settlements. Even more, this trend became intensified in the last two hundred years. With the growth of available power and water production, the presence of a large water artery was the determining factor in the development of a territory.

A parallel course was the development of inland water transport. Prior to the period of the industrial revolution of the 19th century, inland navigation actively used vessels with small displacement. Primarily, this was due to the fact that the pulling forces of small boats at that time were a sail, oars and boatmen pull. Such available power did not allow to increasing the displacement of the ship and their draft to the desired value. As a result the natural dimensions of inland waterways without any additional effort covered current needs.

The situation changed dramatically with the invention of the engine. Increase the power of inland navigation vessels, allowed to increase their loading and draught. In the result, the dimensions of the vessels and their precipitation began to increase and in parallel with them began to grow the volume of track work. The

catalyst of this process was that with the growth in size of the paths and increase of the carrying capacity of the fleet, the cost of water transport traffic fell and the investment attractiveness of water transport grew. In this regard, ship owners began to build ships with the maximum possible displacement and demanded the government to increase the size of the waterways, with the aim of increasing the owners’ profits.

As a result, trunks and over trunks appeared in Russian and European inland water ways. Particularly in the USSR, on the basis of the main rivers of the European part of the country was unified deep water system with guaranteed depth of 4 meters. As a result, vessels of mixed “river-sea” sailing, Moscow became a port of five seas. If at the initial stage, guaranteeing four meters required a moderate investment, over the years, costs began to rise. It was affected by changes in the economic situation of the country, and global warming. Due to the decrease in the water level of rivers, their water balance has changed significantly and breakdowns of guaranteed depths in the key areas of Russian waterways have become common. Attempts of technical intervention in the process by dredging and correctional works, only partially solved the problem but was unable to solve it in principle, as it was before. The reason

for this is that the limiting area for a movement of the fleet is not rivers, but the concrete structures (navigable channels, gateways, etc.). In addition, the trend towards the use of the maximum carrying depths' capacity, has led to the fact, that the dimensions of the fleet construction were close to the dimensions of waterways. As a result, the distance between the bottom and the bottom point of the hull of the vessel was decreased to 20-40 cm.

The reduction in depth under the hull dramatically increased the gradients' of speed acting the bottom. Together with the dynamic nature of the speed impact, the erosion processes both on the bottom and along the banks are also increasing. The intensity of soil erosion from exposure of the vessels, began to change the profiles of river flows, and changes in the dimensions of waterways prevented sustainable navigation on several rivers in Europe. The only way out of this situation was the coating of the bottom and banks of the rivers by concrete panels, which changed the natural waterways into man-made channels.

2. Methods

It is practically proved, that the use of a laboratory experiment at the present stage is impractical, due to the presence of a large-scaled effect. More promising and cheaper method is mathe-

In addition to the negative anthropogenic impact on erosion processes, the compression of the living section of the river complicates the navigation of vessels. Significant increase in the rate of flow of vessels, complex hydrodynamic processes in the area of the hull, the activation of the "piston" and "bank" effects – all these things greatly reduce the security level of vessel traffic.

One way of reducing the above described negative effects and prevention of industrial accidents, is a preliminary, comprehensive study of the dynamics of the interaction of the system "ship – waterway". As a result of the deep, systematic research, even at the stage of preliminary design, the specialists have the opportunity to minimize the negative impact on the environment, and to reduce to zero the probability of possible technogenic accidents. In addition to this, it will be possible to include in the transportation process the additional resources available for the passage of ships on limiting and difficult areas of inland waterways.

matical modeling, based on solution of system differential equations of real fluid motion by Navier-Stokes, supplemented by the equation of continuity:

$$\begin{aligned} \frac{dV_x}{dt} - v \cdot \left(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial y^2} + \frac{\partial^2 V_x}{\partial z^2} \right) &= F_x - \frac{1}{\rho} \cdot \frac{\partial p}{\partial x}; \\ \frac{dV_y}{dt} - v \cdot \left(\frac{\partial^2 V_y}{\partial x^2} + \frac{\partial^2 V_y}{\partial y^2} + \frac{\partial^2 V_y}{\partial z^2} \right) &= F_y - \frac{1}{\rho} \cdot \frac{\partial p}{\partial y}; \\ \frac{dV_z}{dt} - v \cdot \left(\frac{\partial^2 V_z}{\partial x^2} + \frac{\partial^2 V_z}{\partial y^2} + \frac{\partial^2 V_z}{\partial z^2} \right) &= F_z - \frac{1}{\rho} \cdot \frac{\partial p}{\partial z}; \\ \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} &= -\frac{1}{\rho} \cdot \frac{dp}{dt}, \end{aligned} \tag{1}$$

where

t – time;

V_x, V_y, V_z – components of absolute velocity of fluid motion;

F_x, F_y, F_z – components of mass forces;

p – piezometric pressure;

ρ – density;

v – effective viscosity.

Taking into account the fact that the simulation of the movement of the waterways associated with the study of the process of wave formation in two-phase environment (water-air) within the boundaries with complex geometry, for equations (1) we use the discretization of the calculated area with the help of the control volume method (Harlow, 1982). The method consists of the following facts. The calculated area is divided into a number of non overlapping control volumes such that each nodal point is contained in a single control volume. The differential equations (1) are integrated for each control volume. To calculate the integrals piecewise profiles are used, which describe the change in the value between the nodal points. The result is the discrete analogue of the differential equation, which includes the values of these quantities in a few key points.

For tracking the boarder of the water-air environment, we can complete the system of equations (1) with the expression for the passive marker – VOF scalar. With this formal value it is expected to implement subsequently the tracking distribution of water-air environments, calculated for the studied region. According to the recommendations of Hurt and Nichols (1981), the VOF scalar value C will be determined by the following expression:

$$\frac{\partial}{\partial t} C + \nabla C \vec{U} = 0 \quad (2)$$

where: c – has only two values 1- for liquids and 0 – for air.

For the final circuit of the system of equations (1) we must determine with a mathemati-

3. Results

The calculated area is formulated from solid models and filled with calculated cells. At the final stage of forming the model a number of boundary regions are highlighted along the contour of the calculated area (see Figure 3):

- A – the area of marine engines that tells the circular flow speed from the rotation of the screws;
- B – the hull of the ship, in borders of which the rough surface passes the flow the longitudinal velocity of the ship;

cal model to describe the turbulent effects in the fluid stream. Explicitly for their presence in the system equations (1) corresponds to the value ν . According to the modern concepts of the turbulent processes nature, the effective viscosity is an algebraic sum of the kinematic and turbulent viscosity (the latter is also called the coefficient of turbulent exchange). Moreover, if the first value is a constant of the liquid, the theoretical definition of the second value has a large number of opinions. The most proven is the so-called k - ε approach, which will be used further. According to this approach the coefficient of viscosity (the turbulent exchange coefficient) is defined as (Launder, 1974; Rodi, 1974):

$$\nu_T = c_p \cdot \frac{k^2}{\varepsilon} \quad (3)$$

where:

- k – the kinetic energy of turbulence,
- ε – the coefficient of turbulent kinetic energy dissipation.

Taking into consideration the previous experience of such tasks, to model the motion of the river we will use the Reynold hypothesis of turbulence and near-wall (transition) area we will use the logarithmic law (Schlichting, 1968). The further formulation of the mathematical model will be realized in relation to the description (modelling) of the movement of the Volgo-Don type, leaving the Tchaikovsky's gateway. Outer boarders of the calculated flow area and a part of the marine approach canal are formed as a solid geometric CAD model using AUTOCAD system and it is shown in Figure 1 and 2.

- C – the concrete shell of the gateway and approach channel, the rough surface of which regulates not-leaking of flow over its contours;
- D – the output section in the approach channel gateway;
- E – the symmetry region with the neighboring string of the gateway.

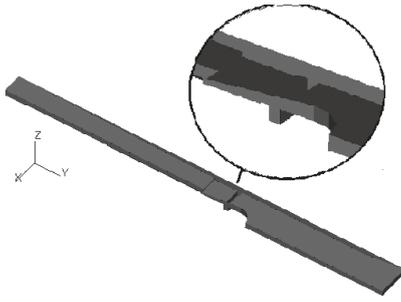


Figure 1. The solid CAD model of the gateway and a part of the approach channel

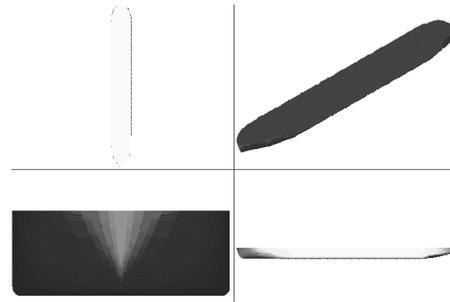
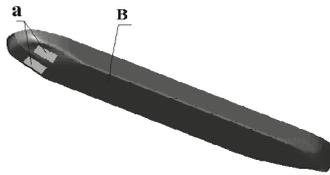
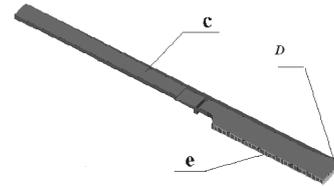


Figure 2. The solid model of the hull of the type Volgabon



a) boundary region on the hull



b) boundary region on the contour of the gateway

Figure 3. Boundary region to the setting of the boundary conditions

For each boundary region the following boundary conditions are created:

$$1. x, y, z \in A \quad \omega = 200 \text{ rpm} ; L = 0.01 : I = 0.01 : VOF = 1 \tag{4}$$

$$2. x, y, z \in B \quad V_x, V_z = 0 ; V_y = V_{ship} ; \kappa = 0 ; \varepsilon = 0 ; dVOF/dn = 0 \tag{5}$$

$$3. x, y, z \in c \quad V_x, V_y, V_z = 0 ; \kappa = 0.0 ; \varepsilon = 0.0 ; dVOF/dn = 0 \tag{6}$$

$$4. x, y, z \in D \quad \frac{dV_n}{dn} = \frac{d\kappa}{dn} = \frac{d\varepsilon}{dn} = \frac{dVOF}{dn} = 0 \tag{7}$$

$$5. x, y, z \in E \quad V_x = 0 ; d\kappa/dn = 0 ; d\varepsilon/dn = 0 ; dVOF/dn = 0 \tag{8}$$

The initial conditions for the model are written as follows:

$$0 < Z < H_k \quad VOF = 1 \quad P = P_{atm} \quad V_x, V_y, V_z = 0 \tag{9}$$

$$Z > H_k \quad VOF = 0 \quad P = 0 \quad V_x, V_y, V_z = 0 \tag{10}$$

To model the behavior of ship's dynamic, let us consider its motion as a rigid body which motions are described by six equations (three – displacement by coordinates, three – rotation around the coordinate axes). Taking into account, that leaving the camera the speed of movement is small, side strokes can be neglected. The rotation around the vertical axis Z can also be neglected (see Figure 4).

The offset by the axis Y is determined by the speed of the hull. The latter will be directly determined by the schedule of the exit from the chamber of the vessel and taken from the experimental data.

For descriptions of the movement of the hull along the axis OZ we will use the Newton's second law in projection on the corresponding axis:



Figure 4. The scheme of the body of the hull movements

$$\vec{\Sigma F} = \vec{m} \cdot \vec{a} = m \cdot \frac{dv}{dt} = m \cdot \frac{d^2y}{dt^2} \tag{11}$$

where:

ΣF – the total force along the Y axis:

m – the mass of the ship

a – vertical projection of the acceleration

The sum of forces consists of two components:

$$\Sigma F = F_A + F_l \tag{12}$$

where:

F_A – Buoyant force

F_l – inertial force

$$a = a_{sh} + g \tag{13}$$

a_{sh} – the acceleration of the ship lift

g – gravitational acceleration (9,81 kg / m*c)

$$F_l = -\lambda * m * a_{sh} \tag{14}$$

where:

λ – coefficient of added masses, described by the formula:

$$\lambda = \frac{B_{sh}}{12 * T_{sh} * \Delta_{sh}} \tag{15}$$

where: B_{sh} – the width of the ship

T_{sh} – the draught of the ship

Δ_{sh} – the reserve under the bottom of the ship

Transforming the equation (13), we get:

$$a_{sh} = \frac{F - m * g}{m * (1 + \lambda)} \tag{16}$$

Modifying the expression (11), we have:

$$\Delta y = V_0 * \Delta t + \frac{a_{sh} * (\Delta t)^2}{2} \tag{17}$$

where: Δy – the offset of the ship for Δt

Δt – step in time

V_0 – the speed of ship’s lifting-lowering on the wave.

The starting point in the description of the ship’s fore and aft pitching is the equation:

$$\Sigma M = J_y * \varphi''(t) \tag{18}$$

where: J_y – the moment of inertia

φ – the roll angle of a ship

ΣM – the total moment, acting on the ship:

$$\Sigma M = M_y - M_l \tag{19}$$

where: M_y – hydrodynamic heeling moment

M_l – attached moment of the ship’s inertia:

$$M_l = J_Y * \varphi''(t) * \lambda \tag{20}$$

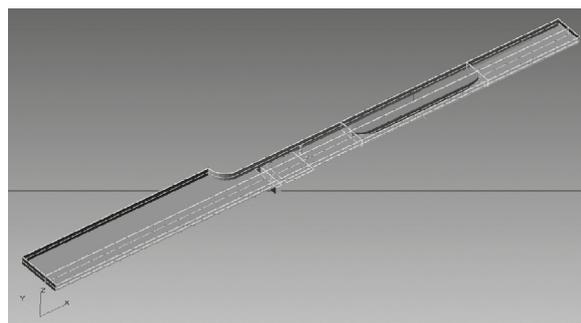
Returning to the equation (18) and integrating it:

$$\varphi''(t) = \frac{M_y}{J_Y * (1 + \lambda)} \tag{21}$$

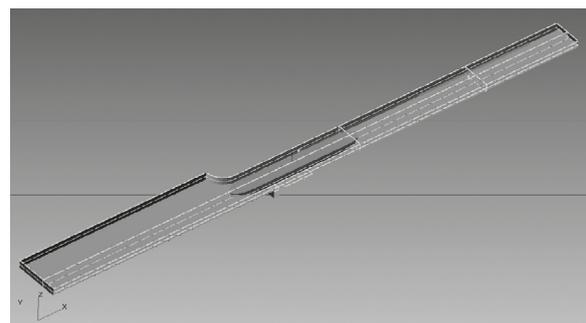
$$\Delta \varphi = \varphi'(t) * \Delta t + \frac{\varphi''(t) * (\Delta t)^2}{2} \tag{22}$$

where: $\Delta \varphi$ – the roll of the ship in time Δt

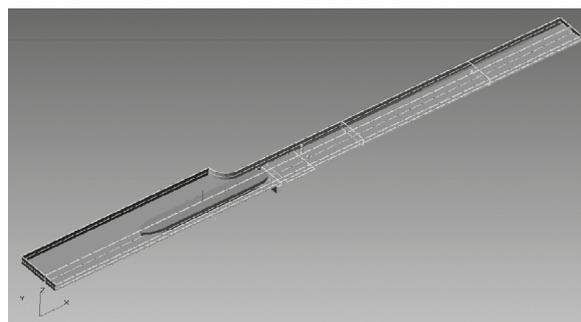
$\varphi'(t)$ – the angular velocity of the ship’s roll



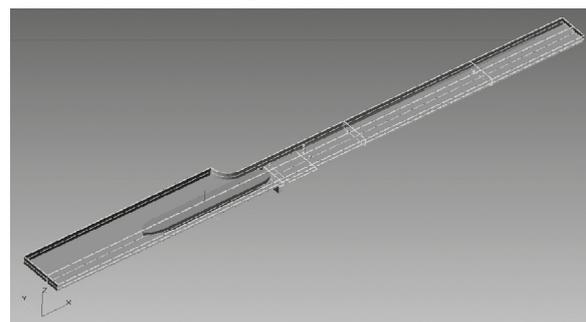
a) initial position



b) the movement along the camera

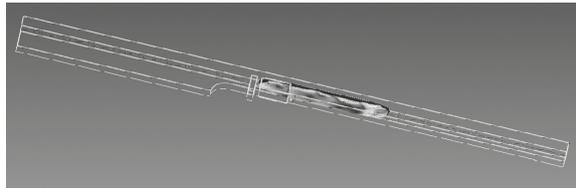


c) the exit from the camera

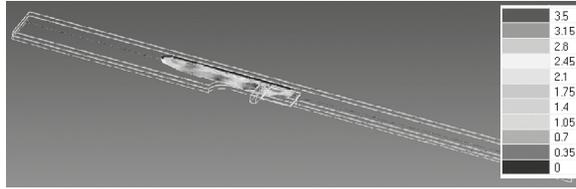


d) the exit from the calculated area

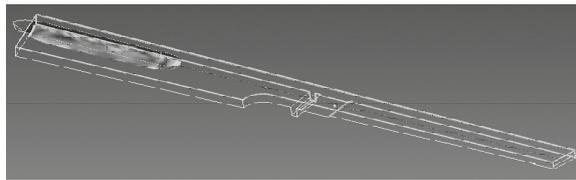
Figure 5. The sequence of deformations of the calculated area during the movement from the camera of the Tchaikovsky’s gateway



a) the vessel in the chamber (beginning of motion):



b) the ship at the exit of the chamber:



c) the vessel moving through the channel:

Figure 6. The velocity [$\text{m}\cdot\text{s}^{-1}$] distribution around the ship's hull.

4. Conclusions

The developed model was tested during the study of eroding velocities around the body of the outgoing Tchaikovsky gateway. Departing from a wall in the middle of the camera, the ship increases its speed to $0.8 \text{ m}\cdot\text{s}^{-1}$. When approaching the threshold (the exit of the camera), the ship accelerates to $0.2 \text{ m}\cdot\text{s}^{-1}$ (see Fig. 6a); when leaving the camera – the speed is $0,65 \text{ m}\cdot\text{s}^{-1}$; moving along the channel the speed is $0.8 \text{ m}\cdot\text{s}^{-1}$.

The initial supply is 40 cm, with a draft of 3.5 m. As a result, when the vessel is moving

The equations (17) and (22) are a part of the calculating procedure, and are calculated after each recalculated step the hydrodynamic variables. In conjunction with a new location of the ship's center of gravity determined by the schedule of the ship, we can determine the new location of the ship in the stream (Figure 5). Then, for the new boundaries of the calculated region, we can calculate the hydrodynamic parameters of the flow. Next the estimated cycle, is implemented again, until the vessel leaves the boundaries of the region.

along the compressed living section, the hull of the vessel experiences complex hydrodynamic processes of flow, provoking wave, roll and trim on the ship's hull. Thus, the erosion speed under the hull, substantially exceeds the speed of the ship and, contrary to the popular view, has an inverse dependence on vessel speed.

This circumstance requires a careful study of the dynamics of the interaction of the system "ship – waterway" in each case to minimize negative consequences in the organization of modern navigation.

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