Leonid A. Turykin<sup>1</sup>, Konstantin M. Berkovich, Leonora V. Zlotina

<sup>1</sup> Lomonosov Moscow State University, Moscow, email: filigorod@list.ru

# The developed (graded) longitudinal profile simulation in the river section (a case study of the Upper Oka River)<sup>1</sup>

**Abstract**: A method of simulating the developed longitudinal profile for a selected river section is proposed. The method is based on the concept of a longitudinal river profile, developed by N.I. Makkaveev. The developed longitudinal profile of the Oka River is calculated from the city of Kaluga to the Beloomut lock/dam. The calculation is performed using an equation of non-uniform flow motion and one of the developed longitudinal profile stage criteria – the constancy of the *QI* product along the river section (*Q* is average multiannual water discharge, *I* is a hydraulic slope). The obtained results show that the developed longitudinal profile of the water surface can serve as an important indicator of the trends of channel processes, not only in theoretical studies but also in solving practical problems of hydrological surveys, for example, when calculating the profile of terminal erosion, assessing the possible lowering of water levels.

Keywords: channel processes, longitudinal river profile, flow transportation capacity

#### 1. The general idea of the longitudinal profile

The longitudinal profile of the water surface reflects the change of the hydraulic gradient (slope) along the river's length – the most important parameter determining the energy of the flow. In the case of uniform fluid motion, the slope reflects the amount of flow energy loss to overcome the hydraulic resistance per unit length. This follows from the equation of uniform fluid motion:

$$\frac{de}{dx} = -\frac{dz}{dx} = -I$$

where de is specific energy.

The hydraulic gradient defines the work of gravity, which is used to overcome the hydraulic resistance of each kilogram of liquid moved over a length of 1 m (Bakheteff, 1934).

The product of QI determines the power used in the stream with water discharge Q per unit length:

$$\frac{dN}{dx} = \gamma QI,$$

where  $\gamma$  – specific weight of liquid.

The longitudinal profile of a river is influenced by many factors – climate conditions, geological and geomorphological structure of the river basin, active tectonics, channel morphology, particle size distribution of sediment, sediment supplies by tributaries, local factors and finally engineering intervention.

In the case of continuous development of vertical erosion, longitudinal profiles of medium and large plain rivers take the form of a concave curve, the slope of which decreases almost logarithmically from the source to the mouth (Fig. 1). At the same time, upon closer examination, the river longitudinal profile of a natural watercourse is usually stepwise due to uneven impact of forming factors in time and space.

<sup>&</sup>lt;sup>1</sup> There is no appropriate term in English to determine the developed longitudinal river profile in Makkaveev's understanding, the closest one is Mackin's "graded" profile (Mackin, 1948)



Figure 1. Longitudinal profile of the Oka River from the source to the mouth

According to conventional notions, the channel incision is terminated when the longitudinal equilibrium profile is generated in which the eroding force of the flow becomes equal to the resistance of the bottom sediment. In these conditions, the vertical deformation of a channel ceases, and the river flow only transports the sediment delivered from the catchment slopes.

### 2. The developed (graded) longitudinal profile

N.I. Makkaveev (1955) expressed doubts about the possibility of reaching the state of equilibrium by a river ("no active" condition) and proposed the concept of a "developed (graded) longitudinal profile", which he understood as a profile matching the stage of development of the channel with "established certain correlations between the gradient and transport capacity of the stream". It is formed in the process of transport capacity along the river with relatively stable climatic and tectonic conditions. In other words, the developed (graded) longitudinal profile (DLP) describes the stage of the channel development with the established relationship between the sediment load and transport capacity of the flow. At the same time, the flow retains the ability to erode the bed and to transport sediment.

There are some properties of DLP following from its definition:

- Longitudinal profile of the water surface retains gradation (steps);
- Longitudinal profile of the channel bottom retains undulation;
- In the case of constant climate, multiannual riverbed deformations disappear, while seasonal deformations persist;
- In the case of steady-state water motion, transport capacity, turbidity, and flow velocity are constant along the length of the river;
- Sediment from the bottom layer does not get into the stream, turbidity is caused by the sediments of the catchment slopes;
- *QI* product tends towards a constant value, the same along the length of the river from the source to the mouth (Berkovich et al., 2016).

#### 3. The purpose and tasks of the study

The purpose of the study was to derive a method of developed (graded) longitudinal profile simulation on a selected river section for practical application in long-term forecasts of longitudinal profile transformation and terminal erosion profile determination.

To achieve this objective, the following tasks were resolved: selection of a river section

to be explored, the actual longitudinal profile plotting, selection of criteria for the developed

longitudinal profile, choosing formulas for calculating DLP.

## 4. The study site and the modern longitudinal profile

The main requirement when selecting a river section to calculate the DLP is the presence of a local base level in the lower reaches, i.e. a site with a minimum hydraulic slope and a minimum level lowering, which can be considered as the beginning of calculations.

The section of the upper Oka from the city of Kaluga to the Beloomut lock/dam, about 300 km long, was selected as a study site. The upper reach of the dam with a crest elevation of 100 m a.s.l. can be considered here as a local base level. The low-head Beloomut lock/dam is located at the 799.6 km from the mouth of the Oka River, at a distance of 50 km below the confluence of the Moscow River. It consists of a collapsible single-span spillway dam, located on the right bank of the channel, and a single-chamber lock on the left bank (Fig. 2). The length of the dam front is 354.3 m. The dam head is 3.5 m and the backwater zone length reaches 60 km at a low water level.



Figure 2. Beloomut lock and dam

An extensive database on the hydrological and riverbed regime has been collected for this site of the Oka River as a result of many years of research. The actual longitudinal profile from l/d Beloomut to v. Tarbushevo was plotted on the basis of one-day levelling made on 06.08.2013, while the profile from v. Tarbushevo to c. Kaluga was created using the project level elevations received from the local waterway service (Fig. 3). The longitudinal profile corresponds to the average multiannual water discharge – 290 m<sup>3</sup>/s in Kaluga and 420 m<sup>3</sup>/s on l/d Beloomut. The lowest bottom elevations are plotted on the same chart. For each point of the longitudinal profile, the mean sediment particle diameter and the bankfull channel width were measured.



Figure 3. Longitudinal profile of the Oka River at the Kaluga-Beloomutin section 2013

The longitudinal profile of the river in this area has a clear stepwise character. Steps correspond to the sites of intense in-stream mining of building materials near the towns of Aleksin, Tarusa (1040–1005 km), Serpukhov (970–950 km), Kashira (920–890 km), Kolomna (860–840 km). In addition, the lower flat section of the longitudinal profile is the result of backwater from l/d Beloomut.

As a criterion for the developed longitudinal profile, the parameter QI is selected, which determines the specific flow velocity. The parameter is defined for the conditions of average multiannual water discharge. Its distribution along the length of the watercourse is presented in Figure 4. Currently, the spread of QI values is large enough.



Figure 4. Distribution of the parameter QI along the section c. Kaluga – l/d Beloomut in 2013

We assume that the value of this parameter at the DLP stage should be approximately the same along the entire length of the watercourse. Since the total amount of flow energy does not change when the base level and upper reach elevation are constant during the longitudinal profile development, the average value of the *QI* parameter may be taken as a simulation constant. To calculate the slope at each point of the DLP longitudinal profile, the basic equation of non-uniform water motion is used:

$$I = \frac{v^2}{C^2 R} + \frac{d}{dx} \left(\frac{v^2}{2g}\right) \tag{1}$$

Chezy coefficient in this case was calculated by Manning's formula:

$$C = \frac{1}{n} R^{1/6} \tag{2}$$

Roughness coefficient – according to the formula of Strickler:

$$n = \frac{0.15}{\sqrt{g}} d^{1/6}$$
(3)

where d is the average particle diameter of bottom sediments.

Since the depth (H) at DLP is unknown, the hydraulic radius (R) is expressed by the width of the channel (B) under the assumption that at the onset of the DLP stage, the width of the channel will not change:

#### 5. Discussion

The result of the DLP calculation is presented in Figure 5. It can be seen that in the lower third of the entire section (the Kashira–Beloomut reach), the actual and developed longitudinal profiles coincide closely enough, even though in the section of intensive in-stream mining in the vicinity of Kolomna town, the actual profile lies 0.5 m below the DLP. Along the reach from Kaluga to Kashira, the actual longitudinal profile exceeds the DLP by 2 m on average (Fig 5). This value determines the potential of vertical erosion and lowering of the water surface.

$$R = H = \frac{Q}{vB} \tag{4}.$$

The average flow velocity ( $\nu$ ) for the average multiannual water discharge at the DLP stage was calculated by the Chezy formula using the parameter QI = 0.012 (average value at the river section from Kaluga to Beloomut with the average multiannual water discharge):

$$v = C \sqrt{\frac{AH}{Q}} \tag{5},$$

where

$$A = QI = 0.012 (6).$$

To exclude the reduction of variable *C*, after substitution 5 by 1, the Chezy coefficient for formula 5 was determined in another way – by R. A. Shestakov's formula (Shestakova, 1963), applicable to floodplain rivers with water surface gradient I = 0.0002- 0.0055, average depth H > 3 m and bankfull channel width B > 100 m:

$$C = 18.5l^{-0.1}($$
(7).

After substitution of 4, 6 and 7 into formula 5, the following will be obtained:

$$v = 2.159 \frac{Q^{1/15}}{B^{1/3}} \tag{8}.$$

To assess the possible vertical deformation of the bottom, the hydraulically feasible average depth for each point of the longitudinal profile was calculated. The calculation was carried out according to formula 4 for the average multi-annual water discharge, velocities calculated for the stage of DLP and the current channel width. The transition from the average depth  $(h_{\rm cp})$  to the maximum depth  $(h_{\rm MAKC})$  was performed by the ratio:



Figure 5. Longitudinal profiles of the water surface of the Oka River: the actual (2013) - 1 and the estimated DLP - 2

$$h_{\text{MAKC}} = \frac{h_{\text{cp}}}{0.7}$$
 (9).

The combined longitudinal profiles of the bottom – estimated and actual – are presented in Figure 5. In the areas of the riverbed quarry, the current bottom elevation is on average 4.5 m below the estimated one for the stage of DLP (the maximum difference is up to 13 m). In the natural (unmodified) sections of the Oka River, the present longitudinal profile of the bottom exceeds the calculated one by 2.2 m on average,



**Figure 6**. Longitudinal profiles of the bottom of the Oka River: the actual one (2013 yr) - 1 and the developed one -2

which equals the difference in the elevation of the corresponding profiles of the water surface. This indicates that in the process of profile development, erosion is expected to spread in the upper half of the studied section, while the lower part will become the ground for sediment deposition.

## 6. Conclusions

- 1. The developed longitudinal profile of the water surface can serve as an important indicator of the trend in the channel processes, not only in theoretical studies but also in solving practical problems of hydrological studies, for example, when calculating the profile of terminal erosion, assessing the possible lowering of water levels.
- 2. The developed longitudinal profile can be calculated with sufficient accuracy using the equation of non-uniform water motion and the correctly selected criterion. In our case, good results were obtained by using the parameter *QI*.
- 3. The coefficient of roughness has a great influence on the results. Obviously, this can be most accurately determined on the basis of hydromorphological field surveys.

## References

Bakheteff B.A., 1934. Hidravlikaot krytyhrusel. Gostransizdat [in Russian].

- Berkovich K.M., Zlotina L.V., Turykin L.A., 2016. The features of Oka-River longitudinal profile development under the anthropogenic disturbances of the riverbed. Geomorphologia 3,3-11 [In Russian with English abstract].
- Mackin J.H., 1948. Concept of the graded river. Geological Society of America Bulletin 59, 463-512.

Makkaveev N.I., 1955. Ruslorekii erozia v eebassejnie. USSR Academy of Science Publishing House [In Russian].

Shestakova R.A., 1963. Opredelenieraskhoda vody pri vysokih urovniyh po uklonu vodnoi poverhnosti I koefficientu C formuly Shezi. GGI 106,71-122 [In Russian].